Chen, Fischer, and Wise Reply: In our Letter [1], we consider two-component systems with one component (molecular species) in its inverted state. We discuss how causality arguments determine the sign of the refractive index n. In particular, we show that negative refraction can be achieved via multiple resonances in the compound dielectric function without the need for a magnetic resonance [1].

The Comment by Mackay and Lakhtakia (ML) [2] focuses on the numerical example presented in Fig. 3(a) of Ref. [1]. ML claim (a) that there are two different representations (I and II) for the phase ϕ_ϵ of the dielectric function $\tilde{\epsilon}(\omega)$ for the example given in Ref. [1] and propose that representation (I) appropriately describes the system. ML further argue (b) that, since $\mathrm{Im}[\epsilon] < 0$ for the wavelengths between 445 and 535 nm, the wave must grow in the $+z$ direction.

We show here that both claims are incorrect and that their proposed solution is unphysical.

(a) As stated in Ref. [1], the medium under consideration is a two-component system. It follows that the dielectric function must therefore have two contributions to its phase angle: $\phi_\epsilon = \phi_{\epsilon 1} + \phi_{\epsilon 2}$, with $-\pi < \phi_{\epsilon 1} \leq \pi$ and $-\pi < \phi_{\epsilon 2} \leq \pi$, which yields $-2\pi < \phi_\epsilon \leq 2\pi$. Representation (I) of ML [2] erroneously restricts the range of ϕ_ϵ to $-\pi < \phi_\epsilon \leq \pi$ and can therefore not describe the example we give in Ref. [1]. While the phase angle of a complex *variable* such as ω in Ref. [1] can always be taken to be its principle value, the phase angle for a *function* of that variable, in this case $\tilde{\epsilon}(\omega)$, is given by the sum of the phases of all its factors $(\omega - \omega_m)$ and $(\omega - \omega_n)^{-1}$, where ω_m and ω_n are the zeros and poles of $\tilde{\epsilon}(\omega)$ [3]. This is used in Ref. [1] to obtain the correct form of ϕ_ϵ, labeled as representation (II) by ML [2]. We note that representation (I) proposed by ML [2] creates additional nonanalytic cuts across the $\mathrm{Re}[\omega]$ axis and leads to a discontinuous refractive index function (see Fig. 1).

(b) While amplification certainly occurs in a medium with $\mathrm{Im}[\epsilon] < 0$, it does *not* necessarily have to be in the $+z$ direction. The "front velocity" must satisfy Einstein causality [4] and point away from the source. The directions of all other quantities, such as group velocity and Poynting vector, follow accordingly. In the case of backward energy flux, for example, the wave is amplified in the $-z$ direction (and the amplitude correspondingly *decays* in the $+z$ direction). The growth of the backward wave in the $-z$ direction in a two-component medium similar to the one we describe in Ref. [1] is shown explicitly by Skaar [5]. The results are independent of the thickness of the medium [5]. In general, $\mathrm{Im}[n] < 0$ signals that the wave amplitude grows along $+z$ and $\mathrm{Im}[n] > 0$ implies that the wave amplitude decays along $+z$. However, the sign of $\mathrm{Im}[n]$ depends on both the signs of $\mathrm{Im}[\epsilon]$ and $\mathrm{Re}[n]$, since $\mathrm{Im}[\epsilon] = 2\epsilon_0 \mathrm{Re}[n]\mathrm{Im}[n]$ [6]. Causality determines the correct sign and leads to a solution with $\mathrm{Re}[n] < 0$ and $\mathrm{Im}[n] > 0$ for the wavelengths between 445 and 535 nm. We stress that the refractive index function in Ref. [1] is causal and continuous at all wavelengths.

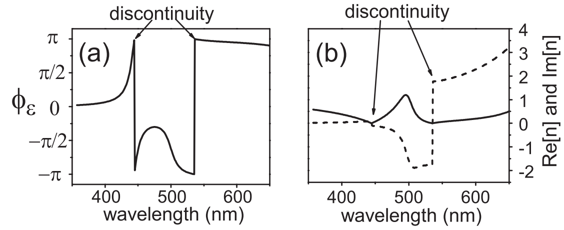

FIG. 1. (a) The phase of the dielectric function ϕ_ϵ ML propose, i.e., representation (I) in Ref. [2]. (b) The corresponding real (solid line) and imaginary (dashed line) parts of the complex refractive index function ML propose in Ref. [2].

These arguments apply equally to the case presented in Fig. 3(b) of Ref. [1].

Considering the above points, we reject the solution proposed by ML [2].

Finally, we take this opportunity to correct a typographical error. As stated in Eq. (9) of Ref. [1], $\beta < 0$. It follows that the expression for β in Eq. (11) of Ref. [1] should have a minus sign. This does not affect the numerical results or any of our conclusions.

Yi-Fan Chen,* Peer Fischer,† and Frank W. Wise‡
Department of Applied and Engineering Physics
Cornell University
Ithaca, New York 14853, USA

Received 2 December 2005; published 17 April 2006
DOI: 10.1103/PhysRevLett.96.159702
PACS numbers: 78.20.Ci, 41.20.Jb, 42.25.Fx

*Electronic address: yc245@cornell.edu
†Electronic address: fischer@rowland.harvard.edu
‡Electronic address: fwise@ccmr.cornell.edu

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